Determining the Competitive Firm’s Short Run Supply Optimal Output Using Excel’s Solver Tool

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Abstract

There has been a recent trend in principles of economics and managerial economics textbooks to incorporate examples of how to solve the standard algebraic models of economics using Excel spreadsheets. However if the examples involve constraints, Excel spreadsheets do not work as well when compared to Excel’s Solver Tool. To illustrate the advantages of using the Solver Tool, we show how to determine the competitive firm’s short run optimal output for the case of cost curves which display all three stages of production. We solve for a particular example, but also show how any problem of this type can be entered into our given Solver set-up. Thus the solution for any particular constrained optimization problem is readily obtained.

Key Words: competitive firm optimum; Excel Solver Tool

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1. Introduction

The inclusion of Microsoft Excel spreadsheet solutions in economic textbooks is a recent and growing trend. Examples of this trend include the questions in Cowen and Tabarrok (2013) and Mankiw (2012). This trend of using spreadsheet in economics courses is also illustrated by Houston (1997), Erfle (2001), Mixon and Tohamy (2002) Caplan (2005) and Schwartz, McPherson and Brastow (2009). The trend of using spreadsheets in courses is evident in other disciplines to as shown by Afrouziyeh et al (2011) and Weber (2007).

In the appendix Cowen and Tabarrok’s chapter 14 after using Excel’ Solver Tool to illustrate price discrimination, they state,

"... we have solved this problem with a combination of economic principles and practical skills ... This combination of principles and practical skills is very powerful and eagerly sought out by employers in a variety of fields."

Unfortunately because of the nonlinearity of typical cost curves and the possibility that the optimal decision for the competitive firm may be to shut down in the short run, it may become difficult to program these types of problem into an Excel spreadsheet per se.

There have been two approaches to avoiding the problem of dealing with constraints. One approach in some principles text books has been to over-simplify the cost curves and use a constant marginal cost curve. Of course, a constant marginal cost curve equals average variable cost at all outputs (MC=AVC). Another approach illustrated by Mixon and Tohamay (2002) is a “black box” approach in which they use an Excel macro in a spreadsheet format to solve for the short run competitive firm supply.3

The first approach unfortunately does not use the curves that principles students are typically taught based on diminishing returns to variable factors of production. The second approach gives the student the correct answer in the case of standard cost curves, but does not allow the student to see and follow how the optimal quantity is found. Our approach, on the other hand, shows all the steps involved in the solution clearly. This means the entire answer can be checked manually even with a hand held calculator if one wishes to do so.

2. Solving for the Competitive Firm Optimum Quantity

The optimum quantity a competitive firm sells in the short-run is found by setting MC equal to price (P). As students in economics know, this is true if and only if price is greater than or equal to minimum AVC. This means, for problems of this type, the first step is always to solve for the quantity at which MC=AVC where AVC is minimized. To illustrate this point, assume the following firm total cost (TC) function applies,

\[ TC = 2Q^3 - 18Q^2 + 60Q + 50. \]

Setting MC = AVC gives,

\[ 6Q^2 - 36Q + 60 = 2Q^2 - 18Q + 60. \]

Solving for Q manually yields Q = 4.5, and plugging back in yields MC = AVC = $19.50.

Since the profit maximizing rule for a competitive firm is P=MC, iff P \( \geq \) minimum AVC, this means the firm will close if the price is less than $19.50 (cannot cover variable costs in the short-run, which are zero if no units (Q) are produced).

3 We call this a “black box” approach as the command to find the optimal result are embedded in the macro and not in the spread sheet cells for students to follow.
This means the short-run competitive firm optimal output, in this example, is generated as follows: setting $MC = P$ gives,

\[ 6Q^2 - 36Q + 60 = P, \]  

which can be rewritten as,

\[ 6Q^2 - 36Q + (60 - P) = 0. \]  

Solving manually for firm supply can then accomplished using the coefficients into the quadratic equation where $a = 6, b = -36, c = (60-P)$. Plugging these coefficients into the quadratic equation formula solving manually for the positive root only, since $MC$ must intersect the price line from below in order to maximize profits, yields,

\[ Q_{firm}^* = \frac{6 + \sqrt{\frac{2}{3}p - 4}}{2}, \]

for $P's \geq$ $19.50$, otherwise, $Q_{firm} = 0$.\(^4\)

Another obvious outcome of interest is the output associated with a normal profit. This means that the $Q$ where $MC = ATC (=\text{min ATC})$ must be determined and then the above steps above repeated to find the normal profit outcome. Step 1: Set $MC = ATC$ and solve for $Q$:

\[ 6Q^2 - 36Q + 60 = 2Q^2 - 18Q + 60 + 50/Q. \]

Obviously, the tedium of solving such problems manually is becoming clear at this point. These manual calculations may provide little benefit in understanding to students.

The purpose of this paper is to show how minimum AVC and minimum ATC can be found quickly and efficiently using Excel’s Solver Tool. This is done in section 3 below.

3. Using Excel Solver Tool to Iterate to the Optimal Output

The optimal output can be found using Excel’s Solver Tool as follows. Enter the following data into a blank Excel sheet:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>enter 1 in B3</td>
<td>Minimize</td>
</tr>
<tr>
<td>2</td>
<td>Q-firm</td>
<td>AVC</td>
</tr>
<tr>
<td>3</td>
<td>quantity</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>st</td>
<td>MC = ATC</td>
</tr>
<tr>
<td>5</td>
<td>st</td>
<td>MC = ATC</td>
</tr>
</tbody>
</table>

Enter 1 in cell B3 (the changing cell). Enter the AVC equation in cell C3 ($=2*B3^2-18*B3+60$) (the set cell). Enter the MC equation in cell B6 ($=6*B3^2-36*B3+60$) (the constraint cell). Then do (make sure cursor is on cell C3): Data (Tab) – Solver (far right under Analysis panel) and fill in the Solver screen as follows:

\[ (6 + \sqrt{9}) / 2 = 9/2 = 4.5 \]

\(^4\) As a check, substituting 19.5 for price ($p$) above yields: $(6 + \sqrt{9}) / 2 = 9/2 = 4.5$
When you select Solve, the following solution is obtained:

Note: always start with 1 entered in cell B3 and increment by one if the Solver solution is Q-Firm = 0 (in cell B3). We are forcing Solver not to select Q-firm = 0, since with the quadratic solution you always end up with either Q-firm = 0 or the positive (in cell B3) Q-firm you want for the supply curve (4.5 > 0) in this case.

For the MC =ATC case, just change the labels appropriately, and enter the ATC equation in cell C3 (just add 50/B3 to the end of AVC equation and Solve.

The following Excel solution verifies that the values for minimum AVC and ATC, respectively are correct.
Corresponding Formulas Used

The Solver method of finding min AVC and min ATC is simple and avoids errors that crop up with the calculator methodology used with most textbooks in principles and intermediate microeconomic course examples for finding the individual competitive firm optimal output in the short-run. Moreover, as the Excel solution sheet above indicates, that Solver has found the correct answer is easily verified using Excel. In addition, using Excel’s drag down methodology, any other prices of interest (covering part of fixed cost, excess profits, and so on) ≥ $19.50 can be added to the price column and then all other values are obtained for that price or prices by simply dragging down the columns.

In fact, once the competitive firm quantities supplied (Q-firm) using Solver for the \( P=MC=AVC \) and \( P=MC=ATC \) cases are determined for a given case, the following Excel methodology can then be applied to solve any problem of this type. This is done by simply replacing the values in cells A2, B2, and C2 with the quadratic equation values (a, b, and c values in the quadratic formula) that apply to the new problem.

\[
Q_{\text{firm}} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}
\]

\[ a \]
\[ ac \]
\[ bb \]
\[ 22 \]
69

As the reader can readily confirm, everything balances correctly in the above Excel output.
Corresponding Formulas Continued

Note after dividing through by 6 to simplify the given values, $a = 1$, $b = -6$ and $c = (10 - 0.16667 \cdot P)$ are produced. The last term [c in cell C2] is entered as a formula (see cell C2 of formula sheet above). This means the entered formula in C2 has to be dragged down through cell C6 in this case for the given prices displayed in D2-D6 above. However, to avoid cluttering, since these are just intermediate calculating values, the values in C3:C6 are hidden. The formulas show, however, when the relevant value for c in cells C3:C6 is clicked on.

The above Excel methodology that applies directly to cost functions that involve all three stages of production and include both variable and fixed costs such as our example cost function, $TC = 2Q^3 - 18Q^2 + 60Q + 50$, can be used as shown above to illustrate the learning outcomes of most interest. These learning outcomes include the firm shutting down in short-run (solution row 2), the price at which the firm produces positive output (solution row 3), minimizing losses in short-run by covering all variable cost and a part of fixed cost (solution row 4), the outcome where only normal profit earned (solution row 5), and the short-run above normal profit case which will attract firm entry in long-run (solution row 6). Most importantly, by using Excel’s Solver Tool to find $P=MC=AVC$ and $P=MC=ATC$ outputs easily to begin with, all competitive firm supply curve problems are then fully and easily solvable in Excel.

4. Summary and Conclusions

We have noted a trend toward using Excel spreadsheet programs to illustrate many of the concepts in principles of economics. The nonlinearity of the typical cost curves and the possibility of the optimal solution being that the firm should shut down in the short run makes the use of Excel spreadsheets more difficult.

Previously, textbook writers have overcome these problems by unrealistically over-simplifying the cost curves or using an Excel macro in a spreadsheet format that literally hides the steps in the solution. Our approach, as shown above, does not have either of these shortcomings. All the steps, and formulas used, in obtaining the short run supply curve of the competitive firm, for example, are clearly shown in the Solver tool methodology. And, most importantly, for any constrained optimization problem, Excel’s Solver tool will quickly and clearly find the correct answer. These answers can also be easily checked.
References


