Interest Rates, Government Bond Sales, and the IS-LM Model

T. Windsor Fields and William R. Hart

Abstract: The conventional wisdom is that interest rates, private investment, and real output are all affected by the government bond sales required to finance a budget deficit. Mainstream macroeconomic textbooks, however, imply no such effect. We demonstrate that the reason for this result is the absence of a financial wealth effect on money demand in the IS-LM model. When such an effect is included, bond sales turn out to have consequences that are consistent with conventional wisdom. In this paper, we sort through these issues, pointing out those areas where the discussion of the economic effects of government bond sales is confusing, incomplete, and, in some cases, incorrect.

Introduction

In recent years there has been heightened concern among professional economists and in the financial community regarding the macroeconomic consequences of substantial and persistent government deficits. This concern has centered on the interest rate effects of government bond sales and the consequent crowding out of private investment that retards economic growth and lowers future living standards. Given the significance of these issues, one would expect macroeconomic textbooks to contain a careful analysis of the effects of debt-financed fiscal actions on the interest rate and a discussion of the conditions under which these bond sales will, or will not, crowd out private spending. Surprisingly, quite the opposite is true. When analyzing

1 Fields: Professor, Department of Economics, James Madison University, Harrisonburg, VA, USA. Phone: 540-568-3097 Fax: 540-568-3010 FIELDSTW@JMU.EDU Hart: Professor, Department of Economics, Miami University, Oxford, OH, USA Phone: 513-529-4352 Fax: 513-529-8047 HARTWR@MUOHIO.EDU (Corresponding Author). The authors thank two anonymous referees for very helpful comments on an earlier version of this paper.

2 In 2000, the federal government ran a budget surplus of $236.2 billion. Following the 2001 recession and the Bush tax cuts, the government budget shifted to a $157.8 billion deficit in 2002 followed by deficits of $377.6 billion in 2003, $412.7 billion in 2004, $318.3 billion in 2005, $248.2 billion in 2006, $160.7 billion in 2007, and $454.8 billion in 2008. With TARP and an $857 billion stimulus package, the 2009 deficit rose to $1.4 trillion. [Source: Historical Budget Data, CBO, January 2010.]

3 In sharp contrast to these concerns is Barro’s [1974, 1979, 1989, 2008] revival of the Ricardian Equivalence Theorem which, in its strongest form, states that deficits do not matter.
the short-run effects of bond-financed fiscal operations, most texts ignore this issue altogether and focus instead on the interest rate effect resulting from the change in real output following the fiscal operation. Our purpose in this paper is to sort through these issues, pointing out those areas where the mainstream view of the economic consequences of government bond sales to finance a rise in government spending (or a tax cut) is confusing and/or incomplete. We employ the IS-LM framework as it provides the most convenient pedagogical and conceptual vehicle for a unified discussion of these issues.

The Basic IS-LM Model

The short-run model in most textbooks, which we refer to as the basic model, is the fixed-price, end-of-period IS-LM framework with wealth, however defined, absent from both the consumption and money demand functions.\(^4\) Algebraically, this model is:

\[
\begin{align*}
\text{(IS)} \quad y &= c(y-t)+i(r)+g \quad (1) \\
\text{(LM)} \quad m &= m^d(y,r), \quad (2)
\end{align*}
\]

where: \(y\) is output, \(t\) is tax revenues, \(r\) is the (real) interest rate, \(c(y-t)\) is desired consumption, \(i(r)\) is desired investment, \(g\) is government purchases of goods and services, \(m^d(y,r)\) is the real demand for money, \(m = M/P\) is the real money supply, and \(P\) is the (fixed) price level. While generally not explicit, certainly implicit in this model is the government budget constraint requiring government purchases of goods and services to be financed by some combination of taxes, bond sales, and money creation:\(^5,^7\)

\[
g = t + \frac{\Delta B^g}{rP} + \frac{\Delta M}{P}. \quad (3)
\]

---

\(^4\) See, for example, Abel, Bernanke, and Croushore [2008], Blanchard [2006], Dornbush, Fischer, & Startz [2008], Froyen [2009], Gordon [2006], and Mankiw [2007].

\(^5\) It is the income variable in the consumption function that distinguishes the Ricardian view from the mainstream view. In the Ricardian view, consumption depends on permanent, or lifetime, income that takes into account current and all future tax liabilities, including future taxes required to pay off the government debt incurred in the current period or required to pay interest forever on that debt if it is not to be paid off.

\(^6\) We assume that each government bond is a consol paying $1/year in interest so that the market price can be written compactly as $1/r. \(B^g\) is the number of government bonds outstanding. To simplify, we ignore interest payments on the outstanding stock of government bonds (i.e., on the accumulated debt).
In the basic model, a rise in $g$ unaccompanied by a change in either $M$ or $t$ necessarily implies debt financing. This result is illustrated in Figure 1. Starting from equilibrium at point A, where $y_1$ is natural output, a bond-financed rise in government expenditures from $g_1$ to $g_2$ shifts the IS curve to the right. The new short-run equilibrium is given by point C.

The standard explanation of the adjustment dynamics is as follows. The rise in $g$ increases aggregate demand and initially causes output to rise by $y_3 - y_1 = \frac{1}{1 - c_y} \cdot [\Delta g]$, where $c_y$ is the marginal propensity to consume. In Figure 1, this corresponds to the movement from point A to point B. At point B, the goods market is back in equilibrium, but the money market is out of equilibrium. The rise in output to $y_3$ increases the transactions demand for money (since $m_y^d = \frac{\partial m^d}{\partial y} > 0$), inducing the private sector to sell bonds in a vain attempt to build its money holdings. The price of bonds falls, and $r$

---

There are two additional budget constraints in the model, the economic consequences of which are frequently overlooked when analyzing the macroeconomic effects of bond-financed fiscal operations. The first is the firms’ budget constraint which requires that investment expenditures be financed by private bond sales: $i(r) = \Delta B^p/rP$, where $B^p$ is the number of private bonds outstanding. We assume private bonds and government bonds are perfect substitutes and that, like government bonds, each private bond is a consol paying $\$1/year in interest. Selling private bonds is the only option available to firms to finance investment because we assume that all firm profits are paid to households as dividends. We also ignore interest payments on the outstanding stock of private bonds. The second constraint, the household budget constraint, requires current consumption plus saving to be financed by current-period disposable income: $c(y - t) + s(y - t) = y - t$. 

---

45
rises, causing investment, hence output, to fall back somewhat from \( y_3 \) to \( y_2 \). The final equilibrium is at point C, where the demand for money has returned to its original level.\(^8\) The important point here is that in the basic model the rise in \( r \) to \( r_2 \) is fully explained by the increase in \( y \) that logically precedes it. The increased stock of government bonds sold to finance the rise in \( g \) has no independent effect on the interest rate.\(^9\)

This brief overview of the effects of a debt-financed rise in \( g \) raises several interesting questions. One, what characteristic of the basic IS-LM model prevents it from capturing an independent effect of government bond supplies on the interest rate? Two, how can the model be altered to capture this independent effect, and what is the economic justification for the alteration? And three, what are the macroeconomic consequences for output, the interest rate, and the crowding out of private expenditures in a model in which increased supplies of government bonds have an independent effect on the interest rate? It is to these questions that we now turn our attention.

**Wealth Effects in the Basic Model**

In any period \( t \), economic agents have a stock of real financial wealth, \( a_t \), equal to the stock inherited from the previous period plus the flow of saving (the change in real wealth) over the period, or:

---

\(^8\) In the Ricardian view, by contrast, a bond-financed (permanent) rise in \( g \) signals higher future taxes equal in present-value terms to the rise in \( g \). Permanent income would, therefore, fall causing consumption to fall, and saving to increase, by the rise in \( g \). The IS curve would remain stationary in Figure 1 leaving output, the interest rate, and investment unchanged.

\(^9\) The context of the one-period result presented above (A\( \rightarrow \)B\( \rightarrow \)C in Figure 1) is that of a fixed-price model in which the level of output is entirely determined by aggregate demand (corresponding to the intersection of the IS and LM curves). In the long run, the excess demand generated by the bond-financed rise in \( g \) raises the price level, shifting the LM curve left until a new long-run equilibrium is reached at point D in Figure 1. The rising price level reduces the real money supply and, once again, causes private sector agents to attempt to reallocate their financial portfolio away from bonds and towards money. The price level, and therefore \( r \), must continue to rise until interest-elastic private spending (investment in the basic model) has been reduced by the same amount as the rise in \( g \). In other words, crowding out is complete in the long run. But still there is no independent effect of government bond sales on \( r \) notwithstanding the fact that the stock supply of these bonds is increasing period-by-period (assuming that the increase in \( g \) is permanent). The additional rise in \( r \) (from \( r_2 \) to \( r_3 \)) is fully accounted for by the rise in \( P \).
\[ a_t = a_{t-1} + s_t \] (4)
in which \( a_{t-1} \) is real financial wealth at the end of the previous period, and \( s_t \) is desired saving in period \( t \). This wealth must be held in some form in private sector asset portfolios. And since in this model the only financial assets are money and bonds (government plus private), there is an implied financial wealth constraint (FWC) requiring the demand for money plus the demand for bonds to equal total real financial wealth. Moreover, since total real financial wealth consists of the real supply of money plus the real supply of bonds, the FWC requires:

\[ \frac{M_t}{P} + \frac{B_t}{rP} = a_t = m^d_t() + b^d_t() \] (5)
in which \( m^d_t() \) is the real demand for money in period \( t \), and \( b^d_t() \) is the real demand for bonds in period \( t \).\(^{10}\)

The FWC is one of several consistency relationships (the others being the various budget constraints) required of all well-specified macroeconomic models. Just as a budget constraint imposes a set of adding-up conditions on the parameters of the flow variables of the model,\(^{11}\) the FWC imposes a set of adding-up conditions on the parameters of the money and bond demand equations (i.e., on the stock variables).\(^{12}\)

Specifically, these adding-up conditions are:

\[ m^d_m + b^d_m \equiv 1 \] (6)
\[ m^d_r + b^d_r \equiv 0 \] (7)
\[ m^d_y + b^d_y \equiv 0 \] (8)

\(^{10}\) The (real) supply of money in period \( t \) is equal to the supply inherited from the previous period, \( M_{t-1}/P \), plus any change in the money supply, \( \Delta M_t/P \), in period \( t \). Similarly, the stock supply of government and private bonds in period \( t \) equals the stock on hand at the end of the previous period, \( B^t_{t-1}/P \), plus the flow supply of government and private bonds in period \( t \). The flow supply of government bonds, \( \Delta B^g/rP \), is determined by the government budget constraint, Equation 3, while the flow supply of private bonds, \( \Delta B^p/rP \) is determined by the firm budget constraint (see footnote 6).

\(^{11}\) For example, the requirement that the marginal propensity to consume plus the marginal propensity to save equals 1.0 is a restriction (or adding-up condition) imposed by the household budget constraint on the consumption-saving decision of households.

\(^{12}\) To our knowledge Brainard and Tobin [1968] were the first to explicitly recognize the adding-up conditions implied by the financial wealth constraint.
in which \( m^d_u = \partial m^d / \partial a_i \), \( b^d_u = \partial b^d / \partial a_i \), \( m^d_r = \partial m^d / \partial r \), \( b^d_r = \partial b^d / \partial r \), \( m^d_y = \partial m^d / \partial y \), 
and \( b^d_y = \partial b^d / \partial y \).

The restrictions implied by equations 6, 7, and 8 enforce consistent behavior on the part of wealth holders in the private sector. Equation 6 requires the private sector to absorb all increases in wealth into their portfolios. Equations 7 and 8 govern the composition of the private sector’s financial wealth portfolio. Specifically, holding wealth constant, any change that, say, raises the demand for money must reduce the demand for bonds by an equal amount. For example, a ceteris paribus decrease in the interest rate will increase the demand for money (since \( m^d_r < 0 \) ) and decrease the demand for bonds by the same absolute amount (since \( b^d_r = -m^d_r \)). A similar conclusion holds for a ceteris paribus change in income. An increase in income, holding wealth constant, raises the demand for money (since \( m^d_y > 0 \) ) at the expense of the demand for bonds (since \( b^d_y = -m^d_y \)).

It may now be apparent why the basic IS-LM model fails to capture an independent bond supply effect. The money demand function in the basic model [i.e., \( m^d_i = m^d(y, r) \)] does not include total financial wealth \( (a_i) \) as an argument. It follows directly that \( m^d_u = 0 \) which, via equation 6, implies \( b^d_u = 1 \). Therefore, an increase in the supply of government bonds required to finance a higher level of \( g \) (or, more generally, a deficit) necessarily generates an equal increase in the demand for bonds. And since the bond supply and bond demand curves shift right by the same amount there cannot be any independent “bond supply effect” on the interest rate in the basic IS-LM model.

---

13 Here we are ignoring, for simplicity, any interest-induced change in the size of the financial wealth portfolio.

14 This adding-up condition may be the most difficult to grasp. An increase in income affects both the desired composition and the desired size of the private sector’s financial wealth portfolio. The composition effect is the effect of the change in income on the desired demand for money and bonds holding wealth constant. This is the effect captured by equation 8. The size effect is the impact of the change in income over time on financial wealth due to the income-induced change in desired saving. Equation 6 ensures that this increase in wealth will be absorbed into private sector asset portfolios.
This outcome is illustrated in Figure 2 in which the economy is initially in equilibrium at point A in panel (a). At this initial equilibrium, aggregate output \( (y_1) \) equals aggregate demand \( (c + i + g) \), and private saving \( (s) \) equals investment \( [i(r_1)] \) plus the government deficit \( (g_1-t_1) \). In the money market, panel (b), equilibrium is at point A where the real demand for money equals the real money supply. In the bond market, panel (c), the real supply of bonds equals the real demand for bonds at point A. The real supply of bonds is the outstanding stock of private and government bonds at the end of the previous period \( (b_{t-1}^{s} = B_{t-1}^{s}/rP_{t-1}) \) plus the real flow supply of bonds during the period \( (\Delta b^{s}) \) required to finance investment \( [i(r_1)] \) and the government deficit \( (g_1-t_1) \).

To isolate the effect of an increased supply of government bonds in the basic IS-LM model, we hold government spending and taxes constant at \( g_1 \) and \( t_1 \), respectively. In addition, we assume that \( g_1 > t_1 \) and that the excess government spending is financed by bond sales. We then ask what happens in the following period (i.e., in \( t+1 \))? Since \( g \) and \( t \) are unchanged in period \( t+1 \), the IS curve remains stationary at its initial (i.e., period \( t \)) position in panel (a). The LM curve must remain stationary as well, for reasons that can be seen in panels (b) and (c). In \( t+1 \), the supply of bonds must increase to finance that period’s investment plus the government deficit. Since investment and the government deficit are the same in period \( t+1 \) as in period \( t \), the bond supply curve will shift to the right by \( \Delta b_{t+1}^{s} = i(r_1) + (g_1 - t_1) \), now passing through point B in panel (c).

As for the demand for bonds, the flow of saving increments wealth, and, since \( s = i(r) + (g - t) \), it follows that the period \( t+1 \) change in financial wealth \( (\Delta a_{t+1}) \) necessarily equals \( \Delta b_{t+1}^{s} \), the change in the total supply of bonds. This increase in wealth must be absorbed into private sector portfolios, and the amount absorbed by increased bond demand is determined by \( b_{t+1}^{d} \) which, in the basic IS-LM model, equals 1. Accordingly, bond demand shifts right in panel (c) by \( \Delta a_{t+1} = s = \Delta b_{t+1}^{s} \) to point B. Since both curves shift right by the same amount, there is no effect on the interest rate. An
increased supply of government bonds to finance a deficit (or, for that matter, private bonds sold to finance investment) is absorbed into private sector portfolios without generating any upward pressure on the interest rate. Finally, there is no impact in the money market, hence no effect on the LM curve. The money supply is exogenous and unchanged in period \( t+1 \), so the money supply curve remains stationary. The money demand curve also remains stationary in period \( t+1 \) because money demand is not affected by wealth accumulation (i.e., because \( m^d_a = 0 \)).

![Graph of IS and LM curves](image.png)

**Figure 2**
Financing a Deficit with Bond Sales in the Basic Model

**Capturing the Direct Impact of an Increased Supply of Bonds**

The simplest modification of the basic model that permits an independent bond supply effect is an expansion of the money demand function to include a second “scale
variable,” namely total real financial wealth \((a_t)\). With this change, the money demand function becomes:

\[
m^d = m^d(y,r,a)
\]  

(9)

in which the partial derivative of real money demand with respect to total real financial wealth \(\frac{\partial m^d}{\partial a} = m^d_a\) is assumed to lie between zero and one. With this specification, any increase in total real financial wealth raises both the real demand for money and the real demand for bonds, but neither increases by as much as \(a_t\). Therefore, an increase in the supply of government bonds to finance a rise in \(g\) (or, for that matter, to finance an on-going deficit) generates an excess supply in the bond market which then exerts independent upward pressure on the interest rate. We refer to the inclusion of total financial wealth in the money demand function as the \textit{portfolio effect}.

At a purely pedagogical level, the argument for including wealth in the money demand function (and therefore implying an independent bond supply effect) is twofold. First, from the very beginning of one’s study of economics students are taught that an increase in the supply of something lowers its price. Yet, in the basic IS-LM model an increase in the supply of bonds fails to lower the price of bonds (i.e., fails to raise the interest rate).\(^{15}\) Teachers of macroeconomics should understand and be able to explain why the absence of wealth as an argument in the money demand function leads to this result, even if they choose not to teach the expanded model. Second, even the most basic theory of finance teaches us that economic agents should diversify their portfolio holdings in order to spread investment risk.\(^{16}\) In the (two asset) IS-LM model the nominal return on money is fixed while the return on bonds is uncertain. Consequently, holding a portion of one’s wealth in the form of money is one way—indeed, the only way in the IS-LM model—in which risk-averse agents can diversify their wealth portfolios and reduce overall risk.\(^{17}\) It should be emphasized that this appeal to

\(^{15}\) Indeed, the authors of this paper have had (perceptive) students make note of this decidedly odd result in class.

\(^{16}\) See Tobin (1958).

\(^{17}\) Friedman (1958) also explores the role of the budget constraint in demand theory as applied to money holdings. His approach emphasizes the importance of permanent income (i.e., wealth) as the appropriate
risk aversion in a non-stochastic model is merely a “heuristic” that teachers can tell students to help them understand why an independent bond-supply effect might exist in the IS-LM model.\(^{18}\)

The expanded IS-LM model now consists of three equations:

(IS) \[ y = c(y-t) + i(r) + g \] \hspace{1cm} (10)

(LM) \[ m = m^d(y, r, a) \] \hspace{1cm} (11)

(TFW) \[ a_t = a_{t-1} + s_t = a_{t-1} + (y-t) - c(y-t) \] \hspace{1cm} (12)

Equation 10 is the basic IS curve, equation 11 is the expanded LM curve, while equation 12 defines total real financial wealth (TFW) as the sum of wealth carried over from period \(t-1\) plus private saving undertaken in period \(t\).\(^{19}\)

The first important difference implied by this expanded IS-LM model is the slope of the LM curve itself. In the basic IS-LM model, the slope of the LM curve is:

\[
\left. \frac{dr}{dy} \right|_{LM} = -\frac{m_y}{m_r} > 0
\] \hspace{1cm} (13)

By contrast, the slope of the expanded LM curve, equation 11, is:\(^{20}\)

\[
\left. \frac{dr}{dy} \right|_{LM} = -\left( \frac{m_y^d + m_y^d [1-c_y]}{m_r^d} \right) > 0
\] \hspace{1cm} (14)

‘constraint variable’ for the demand for money (as well as for other assets). In summarizing this literature, Gordon (2006, pg. 438) states that “the portfolio approach pioneered by both Tobin and Friedman makes the demand for money a function of both income and wealth, not just income.”

\(^{18}\) For example, Keynes’ speculative demand for money is frequently invoked as an explanation for interest elastic money demand in a non-stochastic IS-LM model.

\(^{19}\) An alternate method of defining total financial wealth is the “supply of assets” approach. In this approach \(a_t = a_{t-1} + \Delta m_t^s + \Delta b_t^{s(g)} + \Delta b_t^{s(p)}\), where \(a_{t-1}\) is the total real supply of money and bonds from the previous period, \(\Delta m_t^s\) is the current-period change in the real money supply, and \(\Delta b_t^{s(g)} + \Delta b_t^{s(p)}\) is the current-period change in the total real supply of bonds. While the change in the money supply is exogenous, the change in the real supply of government bonds is determined by the government budget constraint, \(\Delta b_t^{s(g)} = g - \Delta m_t^s\), while the change in the supply of private bonds is determined by the firm budget constraint, \(\Delta b_t^{s(p)} = i(r)\). Substituting from the budget constraints for \(\Delta b_t^{s(g)}\) and \(\Delta b_t^{s(p)}\) into the expression for \(a_t\) given above yields \(a_t = a_{t-1} + (g-t) + i(r)\). Since \(s = (g-t) + i(r)\), it follows that the two approaches are equivalent.

\(^{20}\) In Equation 14, \(c_y\) is the Marginal Propensity to Consume, or \(\partial c/\partial y\).
Comparing these results, we see that the expanded LM is steeper than the basic LM curve. This is because the basic LM curve captures only the impact of a rise in output on the transactions demand for money while the expanded LM captures this effect plus the portfolio effect arising from the output-induced rise (via saving) in total financial wealth.

This is illustrated in Figure 3. LM1 is the basic LM curve, and LM2 is the expanded curve. Now suppose that, starting from point A where the money market is in equilibrium on both curves, there is a $1 rise in output to \( y_2 = y_1 + 1 \) while the interest rate remains unchanged at \( r_1 \). This corresponds to the movement from point A to point B. With LM1, the rise in income increases the transactions demand for money by \( m_y^d \), creating excess money demand equal to \( m_y^d \) at point B. With LM2, however, the increase in income: (1) increases the transactions demand for money by \( m_y^d \) and (2) increases saving and hence total financial wealth by \((1-c_y)\) which increases money demand by \( m_y^d (1-c_y) \). For LM2, then, the excess demand for money at point B is equal to \( m_y^d + m_y^d (1-c_y) \), which clearly is greater than the excess money demand corresponding to LM1. As a result, it takes a larger rise in \( r \) to restore equilibrium at \( y_2 \) on LM2 (\( r_3 \) at point D) than on LM1 (\( r_2 \) at point C). Thus, the expanded LM must be steeper.

![Figure 3](image.png)

The Expanded LM Curve vs. the Basic LM Curve
Fiscal Policy Revisited

Having respecified the IS-LM model in such a way as to capture the impact on financial markets of increased bond supplies, we now revisit the macroeconomic consequences of a bond-financed rise in $g$. Figure 4 pairs the IS curve with both LM curves from Figure 3. The initial short-run equilibrium is at point A where output is $y_1$ and the interest rate is $r_1$. Now suppose that there is a bond-financed rise in government expenditures from $g_1$ to $g_2$ that shifts the IS curve to IS($g_2$). Using the basic LM curve, the new short-run equilibrium will be at point B. Output increases to $y_3$ as the interest rate rises to $r_2$ via the output effect discussed earlier. Using the expanded LM curve, however, the demand for money also rises because of the increase in total real financial wealth, creating additional excess demand in the money market (as well as additional excess supply in the bond market). The *portfolio effect*, then, raises the interest rate beyond $r_2$ to $r_3$ (reducing the rise in output to $y_2$ from $y_3$) as the economy moves to a short-run equilibrium at point C.

![Figure 4](image)

*Figure 4*  
A Bond-Financed Rise in $g$ in the Basic and Expanded Models

Intuitively, the increased supply of bonds required to finance the higher level of $g$ is no longer being offset by an equal increase in bond demand. Consequently, the bond demand curve shifts right by less the bond supply curve, putting additional upward pressure on the interest rate. We may properly call this a “bond supply effect,” and it is this effect that shows up in Figure 4 as the added rise in the interest rate from $r_2$ to $r_3$. 

54
Mathematically, the multiplier for a bond-financed rise in government expenditures in the expanded IS-LM model is:

\[
\frac{dy}{dg} = \frac{1}{(1-c_y) + m^d_y \left( \frac{i_r}{m^d_r} \right) + m^d_a (1-c_y) \left( \frac{i_r}{m^d_r} \right)} = + > 0. \tag{15}
\]

The “portfolio” or “bond supply effect” is the third term in the denominator of equation 15, and it dampens, but does not eliminate, the increase in output in Figure 4.\textsuperscript{21} It is interesting to note that the portfolio effect will cause output to decline in future periods if the bond-financed rise in \( g \) is permanent. To understand why this is so, consider what happens in period \( t+2 \). Government spending is unchanged at \( g_2 \) which means the IS curve remains stationary. The expanded LM curve, however, is not stable. Period \( t+2 \) saving increases total financial wealth and therefore the demand for assets (i.e., the demand for money plus the demand for bonds). Investment, \( i(r) \), plus the government deficit \( (g-t) \) determines the period \( t+2 \) increase in the supply of bonds. And since saving must equal investment plus the government deficit, it follows that:

\[
s_{t+2} = \Delta m^d + \Delta b^d = \Delta a = \Delta b^r = i(r) + (g-t). \tag{16}
\]

With \( m^d_a > 0 \), the increase in bond demand will be less than the increase in bond supply. This puts further upward pressure on the interest rate in \( t+2 \), crowds out additional investment spending, and causes output to fall. Graphically, this effect is captured by an upward shift in the LM curve in period \( t+2 \). This shift continues period after period so long as saving, hence the change in total financial wealth, is positive.

Mathematically, the period \( t+2 \) change in output resulting from the upward shift in the LM curve is:

\[
\frac{dy}{da} = \frac{-m^d_a}{(1-c_y) \left( m^d_a + m^d_r \right) + m^d_y} < 0. \tag{17}
\]

\textsuperscript{21} If \( m^d_a = 0 \), equation 15 reduces to the basic IS-LM multiplier for a bond-financed rise in \( g \).
Equation 17 is unambiguously negative.\textsuperscript{22} So, following a bond-financed rise in \( g \) that increases output in the current period (equation 15), output falls in future periods due to the continued upward pressure on the interest rate (and continued crowding out of private investment) caused by the steadily increasing supply of government bonds required to finance the deficit on a period-by-period basis.\textsuperscript{23,24}

Conclusion

In this paper we have discussed the mainstream view, as it appears in almost all macroeconomic textbooks, of the effect on the interest rate of an increase in the supply of bonds required to finance an increase in government spending. From a pedagogical perspective, we have found much that teachers should find confusing. The basic IS-LM model that is commonly employed implicitly assumes there is no impact on the interest rate from the sale of government bonds via the financial market. The increased supply of government bonds is absorbed into private sector portfolios without requiring any increase in the interest rate. As a result, a deficit arising from an increase in government expenditures has no independent effect on the economy. In other words, deficits do not matter. However, when we expand the IS-LM model to incorporate a financial wealth effect on money demand, we find that deficits do matter, and that they matter in a way that is fully consistent with the concerns expressed by professional economists and the community.

\textsuperscript{22} To determine the period \( t+2 \) impact on output, we rewrote equations 10, 11, and 12 in linear form and solved for the reduced-form equation for output. Since \( m_d^G > 0 \), total financial wealth enters this equation with a one-period lag and a negative effect on output. And since saving increments wealth, output must continue to fall so long as saving is positive. This effect (for one period) is given by equation 17.

\textsuperscript{23} Our analysis is contingent upon the absence of a wealth effect in the consumption function. If government bonds are considered wealth by the private sector, and if wealth has a positive effect on consumption, the IS curve would shift upwards, changing our results. We ignore the wealth effect on consumption for two reasons. One, it has been discussed widely in the literature, and two, it detracts from our primary objective which is to explore the financial market impact of an increased supply of bonds that accompanies debt-financed fiscal operations. A wealth effect on consumption could, of course, offset the future period decline in output arising from the portfolio effect.

\textsuperscript{24} Because the LM curve shifts left as long as saving is positive, saving must equal zero in the long run. And since \( s = i(r) + (g - t) \), it follows that \( i(r) = -(g - t) \) in long-run equilibrium. This also implies a long-run multiplier for bond-financed increases in \( g \) of zero (a result that, interestingly, does not hinge on price flexibility).
References


