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Can Economic Education Improve Entry-Level Math Skills?[1]

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Abstract

Previous studies have shown that mathematics aptitude provides spillover benefits to learning economics principles. However, there is limited evidence of economics instruction developing mathematical skills. This article re-examines the Smyth and Kroncke (2005) study using data from a four-year university. We employ a two-step econometric procedure and decompose our findings according to the type of math question posed to students: traditional algebra, graph-based algebra, and word-problem algebra. Controlling for student characteristics, we find widespread improvements in entry-level math skills among students completing standard courses in introductory economics. These improvements are the largest in traditional algebra skills and in word-problem algebra skills.

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1. Introduction

The archetypical General Education program distinguishes the skills of *formal reasoning* and of *quantitative reasoning* for obvious reasons: the former is the reasoning that entails using operations of formal logic while the latter is the reasoning that entails applying formal logic to data sets (Elrod, 2014). Although there are natural instances of crossover between those two intellectual skills, the standard college curriculum assigns their development to the discrete concept of the *course*.

Thus, despite the arguments articulated by scholars such as Foehl (1993) and Gaze (2014) in favor of designing curricula capable of delivering "formative knowledge content" (Foehl, 1993, p.142) while "providing meaningful engagement in mathematics" (Gaze, 2014, p.3), the compartmentalization of the tasks of acquiring and of applying the tools and operations of formal logic is very much prevalent in the standard college-level curriculum. Alternatives to current practices are put forward by authors such as Madison (2009), who argues for "sharing the responsibility" (Madison, 2009, p.4) of numeracy education. Also, scholars such as Grawe and Rutz (2009) and Pinter (2014) have proposed the deliberate placing of mathematics "in the context of argument" (Grawe and Rutz, 2009, p.3) by weaving formal reasoning into writing assignments and programs.

The social science of economics is considered a partner discipline to mathematics because, in its pursuit of the goal of educating students to "think like an economist" (Siegfried et al., 1991, p.199), it relies heavily on the formal logic of mathematics. A distinctive feature of the professional economist skill set is her ability to strip down any given social phenomenon to its fundamental parts and then describe their logical connections through a model. Moreover, the intellectual partnership between economics and mathematics is constantly deepening. As Wible (2009) bluntly puts it: "Every generation or so, the profession escalates the level and complexity of its mathematical and quantitative tools" to the point where it has become a "mathematical arms race" (Wible, 2009, p.67).

Over the past 25 years, economic education research has documented that some degree of proficiency in mathematics spills over into student performance in economics. Following Smyth and Kroncke (2005), in the study below we posit that, by grounding basic mathematics skills in the social science field of economics, the use that students make of formal reasoning tools becomes more sophisticated. In other words, following Smyth and Kroncke (2005) we argue that an introductory economics education reinforces the mastery of mathematical analytical tools. Our study replicates and extends the Smyth and Kroncke (2005) approach to the measurement of this potential phenomenon by examining data from a four-year (rather than a two-year) institution, by comparing results for both a *weak* and *strict* definition of improvement in math skills, and by decomposing the types of mathematics spillovers into their various subcomponents (i.e., traditional algebra skills, graph-based algebra skills, and word-based algebra skills).

We begin by providing a brief literature review in section two, followed by a description of the data collection process and a discussion of our empirical approach in section three. Then, our empirical results are presented in section four. We conclude our work by discussing our findings and providing some concluding remarks in section five.

2. Prior literature: A brief review

The foundational study of the relationship between mathematics skills and performance in economics principles by Milkman, McCoy, and Brassfield (1995) concludes that competency in introductory-level calculus has a positive and statistically significant effect on grades in both principles of macroeconomics and principles of microeconomics. This basic result is supported by a number of studies published within the first 10 years after the publication of Milkman et al. (1995). For example, a study by Hafer and Hafer (2002) suggests that performance on a pre-course math test serves as an early-warning signal for potential problems in learning economics principles. This result supports their call for principles instructors to use such a test to identify and intervene on behalf of at-risk students. Similarly, Kroncke and Smyth (2003) find that successful completion of a non-remedial math course improves student performance on a subject-specific microeconomics quiz.[3]

Studies by Ballard and Johnson (2004) and Schuhmann, McGoldrick, and Burrus (2005) both find that mathematics aptitude provides spillover benefits to learning in economics principles. The latter of these two studies employs a pre- and post-course math quiz to explore (1) the degree to which students who have greater math aptitude also have a better grasp of basic economics concepts prior to taking a principles of economics course, and (2) the degree to which this math aptitude is correlated with higher economic learning. Data examined in their study indicate that mathematics competency, such as the ability to interpret information presented through a graph, is a significant determinant of learning, over the course of a semester, in principles of economics. Not all studies find spillover benefits such as those discussed above. Laferlöf and Seltzer (2009), for example, explored a natural experiment wherein some students were required to complete a remedial mathematics course prior to entry into principles of economics while others did not face such a requirement. Their results indicate that although performance in secondary school math courses has predictive power on students' performance in economics principles, performance in a remedial math course doesn't impact student performance.

More recently, Arnold and Straten (2012) connect mathematics competency and student motivation in an exploration of the relationship between mathematics competency and success in economics principles. Their results support prior work by linking math proficiency to success in economics principles. They also find that motivation is important, with intrinsic motivation assisting economics principles students in overcoming deficiencies in mathematics preparation. Relatedly, Mallik and Shankar (2016) demonstrate that higher levels of mathematics and economics taken prior to one's university-level principles of economics course are associated with significantly improved student performance in principles of economics. Interestingly, their statistical analysis reveals that prior exposure to economics has a greater impact on performance in principles of economics than does prior exposure to mathematics.

The foundation for the analysis conducted in the present study comes from a research paper by Smyth and Kroncke (2005), which explores the possibility that exposure to economics principles presents spillover benefits to economics principles students in the form of enhanced

[3] Kroncke and Smyth (2003) find that performance in the math course enhances one's performance on an indifference curve analysis quiz.

entry-level mathematics skills. These authors administered a 12-question entry-level math quiz, shown in the Appendix, to 258 economics principles students, both pre- and post-course, at a two-year public college in the U.S. South.[4] The sample analyzed by Smyth and Kroncke produced pre- and post-course mean quiz averages of 7.18 and 8.01, respectively, for a difference of 0.83, which was significantly different from zero at better than the 99 percent level of confidence. Moreover, regression analysis of their data suggests that quiz score improvements are positively and significantly related to prior completion of a non-remedial college math course and the student's age, yet negatively and significantly related to prior completion of a college economics course, the highest level of formal education of one's father, and one's gender, with female student showing less improvement than their male counterparts.

In the sections that follow, we re-examine Smyth and Kroncke's (2005) study using data from a four-year university and a two-step econometric procedure – a hurdle model – that better fits the issue at hand. We test the claim that an introductory economic education course reinforces the mastery of mathematical analytical tools by examining three different categories of questions present in the Smyth and Kroncke (2005) quiz: *traditional* algebra questions (i.e., exclusively employing variables and formulas – devoid of practical application or visual clues); *word-problem* algebra questions (i.e., embedded in economic-related practical problems); and *graph-based* algebra questions (i.e., incorporating a diagrammatic representation of the relationship between two variables).

3. Data and empirical strategy

For this study, the 12-question entry-level math quiz used by Smyth and Kroncke (2005) was administered to 324 economics principles students at a four-year, regional public university in the U.S. South. The quiz includes five *traditional* algebra questions, four *word-problem* algebra questions, and three *graph-based* algebra questions. As in the original study, the quiz was administered on a pre- and post-course basis, and over the course of five consecutive semesters, beginning in fall 2015 and proceeding through spring 2017. The pre-course quiz also included a survey designed to capture relevant demographic data on each student, such as gender, age, employment status, and parents' education levels. Unlike the previous work by Smyth and Kroncke (2005), all of the students were instructed by the same professor using the same pedagogical approach, learning resources, and assessment procedures. The quizzes were scored on a 12-point basis, with each question being worth a single point, and the students were not made aware of any consideration about the *type* of questions they were answering (e.g., *traditional* algebra question). Mean scores are calculated on a course-level basis, and the pre- and post-course means are then compared. The difference between the two means is treated stochastically by using a difference in means test of the null hypothesis that the difference in means is equal to zero. This relatively straightforward course-level approach is then repeated using the pooled sample (n = 324).

[4] Smyth and Kroncke (2005) indicate that these students were taught by two separate instructors, whereby one instructor taught 50.4 percent of the students, while the other instructed the remaining 49.6 percent of the students.

For a more formal analysis of the data provided by the pre- and post-course math quizzes, a hurdle model is developed to examine the determinants of the probability that student *i*'s math skills improve over a semester in principles of economics, and, if improvement is detected, the determinants of the amount of improvement that occurs. The hurdle regression developed by (Mullahy, 1986) treats the count outcome, which in this study is represented by student *i*'s improvement on the math quiz, as emanating from separate statistical processes. The first statistical process involves a binomial distribution determining if a count variable is zero or nonzero, while the second statistical process involves a truncated distribution for count data including all positive counts conditional on nonzero outcomes.

The first hurdle in our study models the tendency for student *i* to show an improvement in mathematics competency during a semester of economics principles. Given that this tendency is a latent continuous variable, the first hurdle is approached by modeling whether or not student *i*'s performance on a post-course math quiz exceeds that on the pre-course quiz. Although this outcome is observable, it represents a binary process wherein the variable of interest, $IMPROVE_i$, is a dummy variable equal to 1 if student *i* displays improvement on the math quiz, and 0 otherwise. A logit model is employed to address the probability that student *i* displays improvement on the quiz (Mullahy, 1986), as denoted below,

$$\Pr(IMPROVE_i=1) = \frac{e^{\beta_0+\beta_1 X_i}}{1+e^{\beta_0+\beta_1 X_i}} \tag{1}$$

where X_i is a vector of exogenous variables, and β_0 and β_1 are parameters to be estimated.

Before turning to the second hurdle, two definitions of improvement on the mathematics quiz are explored in this study. The first is a *weak definition of improvement* whereby $W\text{-}IMPROVE_i$ is coded as a 1 if the difference between student *i*'s pre- and post-course math quiz scores is equal to or greater than 0, and 0 otherwise. Secondarily, we also employ a *strict definition of improvement* whereby $S\text{-}IMPROVE_i$ is coded as a 1 if the difference between student *i*'s pre- and post-course math quiz scores is equal to or greater than 1, and 0 otherwise.

Given that improvement, whether *weakly* or *strictly* defined, is shown over the pre- and post-course math quizzes, the second hurdle explored in this study involves the degree of improvement, as measured by the point differential between the pre- and post-course math quizzes. The dependent variable capturing that differential, $DIFF_i$, is a discrete count, amenable to Poisson regression estimation.[5] Lastly, although the vector of exogenous variables, X_i, shown in (1) above is used to explain the discrete count observations in the second hurdle, it is allowed to have different impacts at each hurdle (Mullahy, 1986).

The vector of exogenous variables, X_i, includes a number of demographic variables, such as $MALE_i$ and AGE_i. The former is a dummy variable equal to 1 if student *i* is male, and 0 otherwise, while the latter is the age, in years, of student *i*. The study by Smyth and Kroncke (2005) suggests that, *ceteris paribus*, male students and older students will exhibit greater improvement on the math quiz than will their female and younger counterparts, respectively. Other demographic regressors included in X_i are $WORK_i$, $FHIGHERED_i$, and $MHIGHERED_i$. The first

[5] See also Winkelmann and Zimmerman (1994), Greene (1997), Cameron and Trivedi (1998), and Kennedy (1998).

of these is a dummy variable equal to 1 if student *i* is employed, and 0 otherwise. $FHIGHERED_i$ is a dummy variable equal to 1 if student *i*'s father earned a four-year college degree, while $MHIGHERED_i$ is a dummy variable equal to 1 if student *i*'s mother earned a four-year college degree. The final demographic regressor, $CCATT_i$, is a dummy variable equal to 1 if student *i* previously attended a community college, and 0 otherwise.

Next, the vector X_i also includes a number of human capital variables, beginning with $HSGPA_i$ and $SAT\text{-}M_i$. The former captures student *i*'s high school grade point average, and the latter is equal to student *i*'s score on the math portion of the SAT entrance exam. Smyth and Kroncke's (2005) analysis omits the first of these two variables, and instead includes each student's college GPA, which is likely an inferior indicator (to high school GPA) of one's academic skills given that many students enroll in economics principles by their second or third college semester. Smyth and Kroncke (2005) also omit the second variable, SAT-M_i, which could lead to biased results given the issue under study, namely improvement of entry-level math skills.

Also included in the human capital portion of X_i are $GEOMETRY_i$ and $PREVECON_i$. The former is a dummy variable equal to 1 if student *i* completed a high school-level course in geometry, and 0 otherwise. The latter is a dummy variable equal to 1 if student *i* completed a prior course in economics at the high school level or beyond, and 0 otherwise. The final two human capital regressors are $MATHNOW_i$ and $ACCTNOW_i$, both of which are included in Smyth and Kroncke (2005). The first is a dummy variable equal to 1 if student *i* is co-enrolled in a college-level mathematics course, and 0 otherwise. The second is a dummy variable equal to 1 if student *i* is co-enrolled in a college-level accounting course, and 0 otherwise.

Next, two important general control variables, PRE_i and $MICRO_i$, are included in X_i, neither of which are tested in the prior study by Smyth and Kroncke (2005). The first, which is equal to student *i*'s score on the pre-course quiz, accounts for the potential scope for improvement that student *i* faces upon completion of the pre-course quiz. The second is a dummy variable equal to 1 if student *i* is enrolled in a principles of microeconomics course, and 0 otherwise. Given that microeconomics principles arguably exposes students to more of the mathematical concepts covered on the quiz, one might expect greater improvement by the students in that course, *ceteris paribus*.

Variable descriptions and summary statistics for all of the variables included in (1) are presented in Table 1. As indicated there, 83.1 percent of all principles students showed *weakly*-defined improvement in entry-level math skills over the course of the semester, while 60.6 percent showed a *strictly*-defined improvement. About 43 percent of the overall sample consists of male students, and the average age of the students is just over 21 years old. About 61 percent of the students were employed at the time they took the principles course, while about 37 percent of students have fathers who earned a four-year college/university degree. This latter statistic compares to about 44 percent of students whose mothers achieved the same level of higher education. Lastly, only about 17 percent of principles students sampled attended community college, thus indicating that our sample is quite different from that examined in Smyth and Kroncke (2005).

In terms of the human capital variables, Table 1 indicates that the mean of all high school GPA's in the sample is just above 3.20 (on a 4-point scale), while the mean of the SAT math scores is 492.5. About 81 percent of students in the sample completed a high school geometry course, while almost 80 percent previously completed an economics course at the high school

Table 1: Variable Descriptions and Summary Statistics

Variable	Description	Mean	Std. Dev
Dependent Variables			
W-IMPROVE$_i$	Dummy variable equal to 1 if the difference between the pre- and post-course quiz scores for student *i* is equal to or greater than 0, and 0 otherwise.	0.831	0.376
S-IMPROVE$_i$	Dummy variable equal to 1 if the difference between the pre- and post-course quiz scores for student *i* is equal to or greater than 1, and 0 otherwise.	0.606	0.489
Demographic Variables			
MALE$_i$	Dummy variable equal to 1 if student *i* is male, and 0 otherwise.	0.431	0.500
AGE$_i$	The age of student *i* at the beginning of the course (years).	21.34	5.49
WORK$_i$	Dummy variable equal to 1 if student *i* is employed, and 0 otherwise.	0.605	0.490
FHIGHERED$_i$	Dummy variable equal to one if student *i*'s father earned a four-year college degree, and 0 otherwise.	0.369	0.483
MHIGHERED$_i$	Dummy variable equal to one if student *i*'s mother earned a four-year college degree, and 0 otherwise.	0.441	0.497
CCATT$_i$	Dummy variable equal to 1 if student *i* previously attended a community college, and 0 otherwise.	0.172	0.378
Human Capital Variables			
HSGPA$_i$	Student *i*'s high school grade point average.	3.205	0.494
SAT-M$_i$	Student *i*'s score on the math portion of the SAT.[#]	492.5	78.44
GEOMETRY$_i$	Dummy variable equal to 1 if student *i* completed a high school geometry course, and 0 otherwise.	0.808	0.394
PREVECON$_i$	Dummy variable equal to 1 if student *i* previously completed an economics course, and 0 otherwise.	0.797	0.403
MATHNOW$_i$	Dummy variable equal to 1 if student *i* is registered for a math course in the current semester, and 0 otherwise.	0.323	0.468
ACCTNOW$_i$	Dummy variable equal to 1 if student *i* is registered for an accounting course in the current semester, and 0 otherwise.	0.339	0.474
Control Variables			
PRE$_i$	Student *i*'s score on the pre-course quiz.	7.920	2.169
MICRO$_i$	Dummy variable equal to 1 if student *i* is currently enrolled in a principles of microeconomics course, and 0 otherwise.	0.486	0.501

[#] For students who completed the ACT entrance exam, scores on the math portion of the ACT are converted to their SAT equivalents using the ThoughCo. conversion table (see http://collegeapps.about.com/od/standardizedtests/a/convertSAT2ACT.htm).

level or beyond. Next, about 32 percent and 34 percent of the sample is, respectively, currently enrolled in an economics course and accounting course. Finally, the means for the additional control variables, PRE_i and $MICRO_i$, are 7.920 points and 48.6 percent, respectively.

4. Empirical results

Summary data for all pre- and post-course quizzes administered over the five separate semesters are presented in Table 2. The total number of students included in the overall sample is 324, with individual course-level sample sizes ranging from 14 to 50 students, for an average of 36. The pre-course quiz mean score for the overall sample is 7.918, while that for the post-course quiz is 8.971. The individual pre-course quiz means range from 7.342 to 8.943, while the individual post-course quiz means range from 8.442 to 9.943. Lastly, all of the differences in means listed in Table 2 exceed 0, ranging from 0.357 to 1.316, with the overall difference in means being 1.053. If treated stochastically, each individual difference in means, with the exception of one of the three for spring 2016 and that for summer 2016, is significantly greater than zero at the 0.051 level or better. The difference in means of 0.734 for the aforementioned spring 2016 score is significantly greater than 0 at the 0.120 level. The difference in means of 0.357 for summer 2016 is significantly greater than 0 at only the 0.330 level, likely as a result of the small individual sample size. Moreover, the difference in means for the pooled sample of 1.053 is significantly greater than zero at better than the 99 percent level of confidence.

Table 2: Quiz Results and Difference in Means Tests: All Questions

Course	Semester	*n*	Pre-Course Mean	Post-Course Mean	Diff. in Means	*p*-value
Principles of Microeconomics	Fall '15	29	7.448	8.724	1.276	0.007
Principles of Macroeconomics-A	Spring '16	30	8.033	8.767	0.734	0.120
Principles of Macroeconomics-B	Spring '16	38	7.342	8.658	1.316	0.005
Principles of Microeconomics	Spring '16	36	7.639	8.944	1.305	0.002
Principles of Microeconomics	Summer '16	14	8.929	9.286	0.357	0.330
Principles of Microeconomics	Fall '16	49	7.714	8.918	1.204	0.004
Principles of Macroeconomics-A	Spring '17	43	7.628	8.442	0.814	0.051
Principles of Macroeconomics-B	Spring '17	50	8.220	9.240	1.020	0.012
Principles of Microeconomics	Spring '17	35	8.943	9.943	1.000	0.012
Pooled Sample	All	324	7.918	8.971	1.053	0.000

Next, quiz results decomposed according to the category of question included in the Smyth and Kroncke (2005) quiz are presented in Tables 3a through 3c. First, we note the improvement in *traditional* algebra skills (Table 3a), as measured by the difference in means between pre-course and post-course quiz scores, in all but one of the course sections. If treated stochastically, six course sections report significant differences at the 0.05 level or better and two additional course sections report significant differences at the 0.10 level or better. Across course sections, there is a 0.497-point mean gain (approximately 14%) in quiz scores associated with *traditional* algebra skills. This average gain is even more remarkable given the aforementioned fact that pre-course quiz scores in this particular category of algebra skills suggest an *ex-ante* high level of student proficiency. Second, we note the general lack of improvement in *graph-based* algebra skills (Table 3b), as measured by the difference in means between pre-course and post-course quiz scores. If treated stochastically, only two course sections report significant

Table 3a: Quiz Results and Difference in Means Tests: Traditional Algebra Questions

Course	Semester	*n*	Pre-Course Mean	Post-Course Mean	Diff. in Means	*p*-value
Principles of Microeconomics	Fall '15	29	3.379	4.103	0.724	0.001
Principles of Macroeconomics-A	Spring '16	30	3.600	4.067	0.467	0.050
Principles of Macroeconomics-B	Spring '16	38	3.421	4.132	0.711	0.001
Principles of Microeconomics	Spring '16	36	3.361	4.111	0.750	0.002
Principles of Microeconomics	Summer '16	14	4.071	4.357	0.286	0.172
Principles of Microeconomics	Fall '16	49	3.633	3.918	0.285	0.096
Principles of Macroeconomics-A	Spring '17	43	3.628	4.047	0.419	0.022
Principles of Macroeconomics-B	Spring '17	50	3.700	4.180	0.480	0.005
Principles of Microeconomics	Spring '17	35	3.971	4.314	0.343	0.057
Pooled Sample	All	324	3.617	4.114	0.497	0.000

Note: There were five *traditional* algebra questions included in the quiz.

Table 3b: Quiz Results and Difference in Means Tests: Graph-Based Algebra Questions

Course	Semester	*n*	Pre-Course Mean	Post-Course Mean	Diff. in Means	*p*-value
Principles of Microeconomics	Fall '15	29	1.000	1.069	0.069	0.371
Principles of Macroeconomics-A	Spring '16	30	1.233	1.233	0.000	—
Principles of Macroeconomics-B	Spring '16	38	1.026	1.079	0.053	0.392
Principles of Microeconomics	Spring '16	36	1.306	1.306	0.000	—
Principles of Microeconomics	Summer '16	14	1.500	1.357	−0.143	0.682
Principles of Microeconomics	Fall '16	49	1.082	1.388	0.306	0.035
Principles of Macroeconomics-A	Spring '17	43	1.209	1.442	0.233	0.082
Principles of Macroeconomics-B	Spring '17	50	1.360	1.460	0.100	0.258
Principles of Microeconomics	Spring '17	35	1.371	1.571	0.200	0.147
Pooled Sample	All	324	1.216	1.336	0.120	0.029

Note: There were three *graph-based* algebra questions included in the quiz.

differences at the 0.10 level or better. This finding might be somehow perplexing given the abundant use that standard courses in introductory economics make of diagrammatic representations of the relationship between two variables (e.g., the supply and demand model). Having said that, it is important to emphasize that these diagrams are generally employed to illustrate the nature of a relationship between two variables (i.e., direct or inverse) and do not – most frequently – attempt to represent any particular functional relationship (i.e., linear vs. exponential). Last, we report statistically significant improvements in *word-problem* algebra skills (Table 3c), present in six out of the nine course sections. This finding gives credence to the ongoing drive by the mathematical education profession (see Ganter and Haver, 2011) to embed their abstract formal reasoning instruction in the specific context of an applied field such as economics.

Table 3c: Quiz Results and Difference in Means Tests: Word-Problem Algebra Questions

Course	Semester	*n*	Pre-Course Mean	Post-Course Mean	Diff. in Means	*p*-value
Principles of Microeconomics	Fall ‘15	29	2.621	3.000	0.379	0.068
Principles of Macroeconomics-A	Spring ‘16	30	2.567	2.633	0.066	0.406
Principles of Macroeconomics-B	Spring ‘16	38	2.421	2.868	0.447	0.035
Principles of Microeconomics	Spring ‘16	36	2.389	3.083	0.694	0.001
Principles of Microeconomics	Summer ‘16	14	2.786	2.786	0.000	—
Principles of Microeconomics	Fall ‘16	49	2.510	2.918	0.408	0.029
Principles of Macroeconomics-A	Spring ‘17	43	2.233	2.419	0.186	0.225
Principles of Macroeconomics-B	Spring ‘17	50	2.540	2.980	0.440	0.016
Principles of Microeconomics	Spring ‘17	35	2.971	3.286	0.315	0.076
Pooled Sample	All	324	2.531	2.889	0.358	0.000

Note: There were four *word-problem* algebra questions included in the quiz.

Results from two hurdle Poisson models are presented in Table 4. The first of these employs the *weak* definition of math skills improvement. According to the estimates regarding the first hurdle in column (1a) of the table, the control for performance on the pre-course quiz is, as expected, both negatively and significantly related to the probability that student *i* shows improvement in math skills over the course of a semester in economics principles. Also, of note in Table 4 is the finding that none of the demographic variables is significantly related to the probability that student *i* exhibits improvement in math skills as a result of exposure to economics principles. This is not quite the case in terms of the human capital variables, as one’s stock of mathematics capital, as measured by *SAT-M*, is positively and significantly (at the 0.01 level) related to the probability that improvement in one’s entry-level math skills is shown.

The results for the second hurdle using the *weak* definition of improvement are presented in column (2a) of Table 4. Here, the control for pre-course test score is negatively related to the amount of improvement in one’s math skills shown over the course of a semester in economics principles. In this case, however, if improvement in math skills is shown, student *i*’s high school GPA is positively and significantly (at the 0.05 level) related to the magnitude of that improvement. Similarly, co-enrollment in a college-level mathematics course is positively and significantly (at the 0.05 level) related to the magnitude of math improvement, given that some improvement, at least *weakly* defined, is shown.

The second set of results in columns (1b) and (2b) of Table 4 tackles the *strict* definition of math skills improvement. Controlling for one’s score on the pre-course quiz again appears to be a relevant feature of the hurdle model, as that score is both negatively and significantly (at the 0.01 level) related to the probability that student *i* shows improvement in math skills during exposure to university-level economics principles. As in the *weak* definition case, none of the demographic variables exhibits a significant relationship to the probability that math skills improvement occurs. Also, as in the *weak* definition analysis, one’s score on the math portion of the SAT is positively and significantly (at the 0.01 level) related to entry-level math improvement, as *strictly* defined. In this case, however, both high school grade point average and current enrollment in a college-level math course are positively and significantly (at the 0.10 level)

Table 4: Hurdle Poisson Results

Improvement:	*Weakly Defined*		*Strictly Defined*	
	Hurdle 1a Logit	Hurdle 2a Poisson	Hurdle 1b Logit	Hurdle 2b Poisson
Variables	(1a)	(2a)	(1b)	(2b)
Constant	−4.39	0.04	−1.07	0.03
	(−1.32)	(0.04)	(−0.49)	(0.03)
Demographic Variables				
MALE	0.53	−0.01	0.32	−0.1 e−2
	(1.33)	(0.03)	(1.02)	(−0.01)
AGE	0.19	0.01	0.01	0.01
	(1.52)	(0.16)	(0.17)	(0.16)
WORK	0.38	−0.02	0.30	−0.02
	(0.95)	(−0.12)	(0.94)	(−0.11)
FHIGHERED	0.44	−0.09	−0.09	−0.09
	(0.92)	(−0.53)	(−0.25)	(−0.51)
MHIGHERED	−0.02	0.13	0.25	0.13
	(−0.05)	(0.83)	(0.74)	(0.81)
CCATT	0.80	0.18	0.52	0.18
	(0.99)	(0.83)	(0.99)	(0.84)
Human Capital Variables				
HSGPA	0.60	0.40‡	0.76†	0.40‡
	(1.15)	(2.12)	(1.84)	(2.13)
SAT-M	0.01*	0.7e−3	0.01*	0.7e−3
	(3.21)	(0.54)	(2.60)	(0.54)
GEOMETRY	0.05	0.17	−0.38	0.17
	(0.09)	(0.74)	(−0.85)	(0.76)
PREVECON	−0.66	0.06	−0.24	0.06
	(−1.08)	(0.30)	(−0.56)	(0.30)
MATHNOW	0.27	0.34‡	0.58†	0. 34‡
	(0.64)	(2.21)	(1.74)	(2.20)
ACCTNOW	0.33	0.07	0.53	0.07
	(0.70)	(0.46)	(1.51)	(0.44)
Control Variables				
PRE	−0.66*	−0.22*	−0.56*	−0.22*
	(−4.94)	(−4.52)	(−5.50)	(−4.53)
MICRO	0.40	−0.05	−0.14	−0.05
	(1.01)	(−0.33)	(−0.48)	(−0.33)
n	240	220	240	147
LL	−93.0	−326.8	−137.0	−187.0

Notes: Numbers in parentheses are *t*-ratios. *(‡)[†] denote the 0.01(0.05)[0.10] levels of significance, respectively.

associated with improvement in math skills over the course of a semester in economics principles. In fact, the results suggest that a marginal increase in high school GPA results in an increase in the probability of strict improvement in the student's math skills by 0.148 (i.e., [∂Pr(*S*-IMPROVE=1)/∂HSGPA] = 0.148), while co-enrollment in a college-level math course increases the probability of strict improvement in the student's math skills by 0.112 (i.e., [∂Pr(*S*-IMPROVE=1)/∂MATHNOW] = 0.112).

Lastly, the results for the second hurdle using the *strict* definition of improvement are presented in column (2b) of Table 4. As before, student *i*'s score on the pre-course quiz is negatively and significantly (at the 0.01 level) related to the magnitude of a student's improvement in math skills with exposure to economics principles. Moreover, as with the first hurdle, a student's high school GPA and co-enrollment in a college-level math course also increase the magnitude of the improvement in math skills over the course of a semester in economics principles. These two variables are both significant at the 0.05 level. Once again, none of the demographic variables shows a significant relationship to the amount of math skills improvement exhibited.

Next, hurdle Poisson results decomposed according to the category of question included in the Smyth and Kroncke (2005) quiz are presented in Tables 5a through 5c. Table 5a presents the results for *traditional* algebra questions. Male students are significantly more likely than female students to exhibit at least some improvement, weakly defined, in math skills over the course of a semester of economics principles. Also, holding constant one's score on the pre-test, which is negative and significant in the model, one's high school GPA is both positively and significantly related to the probability that one shows some math skills improvement. In terms of the magnitude of one's improvement, weakly defined, shown in column (2a) of Table 5a, only one's pre-test score, which is negatively signed, as expected, is significant.

The second set of results shown in Table 5a are based on the strict definition of improvement in one's math skills. In terms of the first hurdle, shown in column (1b), not only is one's pre-test score both negatively signed and significant, as expected, but having a father with a four-year college degree reduces the probability that one shows math skills improvement over the course of a semester of economics principles. As before, one's high school GPA and one's score on the math portion of the SAT are both positively and significantly related to the probability one shows math skills improvement, as strictly defined. Lastly, one's pre-test score is, as shown in column (2b) in Table 5a, the only significant variable in terms of explaining the magnitude (i.e., the second hurdle) of one's math skills improvement.

The results in Table 5b focus on performance across the *graph-based* algebra questions. There, in terms of exhibiting at least some weakly-defined improvement, the results in column (1a) indicate that one's pre-test score is negatively signed and significant, while one's score on the math portion of the SAT is positively signed and significant. Interestingly, students who maintain employment are more likely to show some math skills improvement, as are those who are co-enrolled in a college-level math course. Surprisingly, students who completed a geometry course in high school face a lower probability of showing improvement in math skills, as weakly defined. In terms of the second hurdle shown in column (2a), only one's high school GPA, which is positive, as expected, is significant in explaining the magnitude of one's math skills improvement.

Table 5a: Hurdle Poisson Results, Traditional Algebra Questions

Improvement:	*Weakly Defined*		*Strictly Defined*	
	Hurdle 1a Logit	Hurdle 2a Poisson	Hurdle 1b Logit	Hurdle 2b Poisson
Variables	(1a)	(2a)	(1b)	(2b)
Constant	−2.19	2.92	2.14	2.93
	(−0.74)	(1.10)	(0.90)	(1.11)
Demographic Variables				
MALE	0.72†	0.19	0.43	0.19
	(1.72)	(0.50)	(1.28)	(0.51)
AGE	0.05	−0.08	−0.04	−0.08
	(0.55)	(−0.89)	(−0.58)	(−0.88)
WORK	0.04	−0.29	−0.09	−0.30
	(0.11)	(−0.82)	(−0.26)	(−0.85)
FHIGHERED	−0.13	−0.12	−0.74†	−0.13
	(−0.27)	(−0.24)	(−1.92)	(−0.26)
MHIGHERED	0.02	0.10	0.40	0.11
	(0.03)	(0.25)	(1.13)	(0.27)
CCATT	0.88	0.49	0.23	0.49
	(1.06)	(0.98)	(0.43)	(0.97)
Human Capital Variables				
HSGPA	1.09‡	−0.39	1.13*	−0.39
	(2.08)	(−0.81)	(2.57)	(−0.81)
SAT-M	0.4e−2	0.3e−2	0.4e−2†	0.3e−2
	(1.37)	(1.01)	(1.70)	(0.99)
GEOMETRY	1.11‡	−0.29	0.20	−0.28
	(2.13)	(−0.66)	(0.42)	(−0.64)
PREVECON	−0.10	0.28	−0.46	0.27
	(−0.18)	(0.53)	(−1.03)	(0.51)
MATHNOW	−0.33	0.39	0.36	0. 39
	(−0.80)	(1.06)	(1.02)	(1.05)
ACCTNOW	0.58	0.58	−0.09	0.59
	(1.17)	(1.49)	(−0.23)	(1.50)
Control Variables				
PRE	−1.02*	−0.76*	−1.90*	−0.76*
	(−3.67)	(−4.19)	(−6.89)	(−4.20)
MICRO	0.01	−0.20	−0.06	−0.20
	(0.02)	(−0.60)	(−0.17)	(−0.59)
n	240	240	240	113
LL	−89.5	−235.1	−121.3	−68.6

Notes: Numbers in parentheses are *t*-ratios. *(‡)[†] denote the 0.01(0.05)[0.10] levels of significance, respectively.

Table 5b: Hurdle Poisson Results, Graph-Based Algebra Questions

Improvement:	*Weakly Defined*		*Strictly Defined*	
	Hurdle 1a Logit	Hurdle 2a Poisson	Hurdle 1b Logit	Hurdle 2b Poisson
Variables	(1a)	(2a)	(1b)	(2b)
Constant	−0.60	4.21	5.35	5.92
	(−0.22)	(0.35)	(1.61)	(0.73)
Demographic Variables				
MALE	−0.16	−1.36	0.21	−1.50
	(−0.42)	(−1.43)	(0.48)	(−1.58)
AGE	−0.06	−0.17	0.01	−0.22
	(−0.71)	(−0.53)	(0.09)	(−0.89)
WORK	1.00‡	−1.13	0.65	−1.22
	(2.54)	(−1.39)	(1.44)	(−1.56)
FHIGHERED	0.36	−0.94	−0.11	−0.90
	(0.77)	(−0.90)	(−0.22)	(−1.03)
MHIGHERED	−0.03	0.39	0.07	0.48
	(−0.06)	(0.34)	(0.14)	(0.46)
CCATT	−0.08	0.23	0.04	0.33
	(−0.12)	(0.21)	(0.06)	(0.33)
Human Capital Variables				
HSGPA	0.10	1.92†	0.91†	1.86†
	(0.21)	(1.91)	(1.65)	(1.90)
SAT-M	0.9e−2*	−0.01	0.01‡	−0.01
	(2.90)	(−0.88)	(2.34)	(−1.47)
GEOMETRY	−1.32‡	0.27	−0.77	0.16
	(−2.08)	(0.19)	(−1.32)	(0.18)
PREVECON	0.53	−1.50	−0.44	−1.66
	(1.01)	(−0.75)	(−0.79)	(−1.30)
MATHNOW	0.74†	1.31	1.01‡	1. 39
	(1.68)	(1.36)	(2.19)	(1.65)
ACCTNOW	0.45	1.23	0.43	1.40
	(1.01)	(1.19)	(0.93)	(1.54)
Control Variables				
PRE	−1.35*	−14.97	−2.83*	−18.87
	(−4.18)	(0.03)	(−7.35)	(−0.00)
MICRO	0.21	−1.57	−0.14	−1.74
	(0.53)	(−1.27)	(−0.33)	(−1.46)
n	240	240	240	63
LL	−93.3	−160.9	−79.8	−22.7

Notes: Numbers in parentheses are *t*-ratios. *(‡)[†] denote the 0.01(0.05)[0.10] levels of significance, respectively.

In the strict definition case shown in columns (1b) and (2b) of Table 5b, one's high school GPA and the score on the math portion of the SAT are positively and significantly related to the probability one shows some math skills improvement. Additionally, one's pre-test score is negative and significant, while co-enrollment on college-level math is positive and significant. Only one of these variables – *HSGPA* – is, however, significant in the second hurdle.

Hurdle Poisson results for *word-based* algebra questions are presented for both the weakly and strictly defined cases in Table 5c. In the former case, both one's high school GPA and one's score on the math portion of the SAT are positively and significantly related to the probability one exhibits math skills improvement. Also, one's pre-test score is again negative and significant in this hurdle. In terms of the second hurdle, having a father with a four-year college degree increases the magnitude of math skills improvement over the course of a semester of economics principles, while having a mother with the same level of education attainment decreases this magnitude. Additionally, one's pre-test score is negatively and significantly related to the magnitude of weakly-defined improvement, while males exhibit significantly less weakly-defined improvement than do females.

Finally, hurdle Poisson results for the strictly defined case across the *word-based* algebra questions are presented in the final two columns of Table 5c. In the first hurdle, only one's pre-test score and one's score on the math portion of the SAT have the expected signs and are significant. In terms of the second hurdle shown in column (2b), having a father with a four-year college degree increases the magnitude of math skills improvement over the course of a semester of economics principles, while having a mother with the same level of education attainment decreases this magnitude. Moreover, as in the weakly-defined case, one's pre-test score is negatively and significantly related to the magnitude of strictly-defined improvement, while males exhibit significantly less strictly-defined improvement than do females.

5. Discussion and conclusion

This paper documents widespread improvements in entry-level math skills among students completing standard courses in introductory economics. These improvements are the largest in *traditional* algebra skills (i.e., solving problems devoid of practical application or visual clues – working exclusively from variables and formulas) and in *word-problem* algebra skills (i.e., identifying and solving problems embedded in economic-related practical problems). Perhaps unexpectedly, *graph-based* algebra skills (i.e., solving problems illustrated by a diagrammatic representation of the relationship between two variables) record smaller improvements.

We also report that human capital variables such as high school GPA, the score in the mathematics portion of the SAT, and concurrent enrollment in a mathematics course contribute to the improvement in mathematics skills over the course of a semester completing a principles of economics course. More interestingly, though, we find that – even when controlling for those factors – students with weaker *ex-ante* mathematical skills improve the most. This finding is reported for both *weak* and *strict* definitions of improvement and it is not associated with student characteristics such as gender, age, working status, educational attainment of the parents, or previous enrollment in a community college.

Table 5c: Hurdle Poisson Results, Word-Based Algebra Questions

Improvement:	*Weakly Defined*		*Strictly Defined*	
	Hurdle 1a Logit	Hurdle 2a Poisson	Hurdle 1b Logit	Hurdle 2b Poisson
Variables	(1a)	(2a)	(1b)	(2b)
Constant	−8.41*	−2.53	−1.60	−2.61
	(−2.58)	(−0.80)	(−0.65)	(−0.82)
Demographic Variables				
MALE	0.28	−0.83†	0.30	−0.82†
	(0.66)	(−1.69)	(0.93)	(−1.67)
AGE	0.16	0.02	0.10	0.02
	(1.50)	(0.22)	(1.37)	(0.22)
WORK	0.15	0.36	0.16	0.37
	(0.34)	(0.86)	(0.46)	(0.88)
FHIGHERED	−0.34	0.80†	−0.15	0.79†
	(−0.68)	(1.68)	(−0.40)	(1.67)
MHIGHERED	0.29	−1.05‡	0.30	−1.05‡
	(0.60)	(−2.21)	(0.81)	(−2.22)
CCATT	0.90	−1.41	−0.12	−1.40
	(1.06)	(−1.26)	(−0.23)	(−1.26)
Human Capital Variables				
HSGPA	1.96*	0.58	0.24	0.60
	(3.36)	(1.42)	(0.53)	(1.45)
SAT-M	0.7e−2‡	0.2e−2	0.01‡	0.2e−2
	(2.17)	(0.49)	(1.97)	(0.49)
GEOMETRY	0.47	0.34	0.15	0.36
	(0.81)	(0.57)	(0.32)	(0.61)
PREVECON	−0.04	0.05	−0.48	0.05
	(−0.06)	(1.12)	(−1.04)	(0.12)
MATHNOW	0.75	0.45	0.41	0.43
	(1.61)	(1.15)	(1.12)	(1.10)
ACCTNOW	0.49	−0.09	0.29	−0.05
	(1.03)	(−0.20)	(0.79)	(−0.12)
Control Variables				
PRE	−1.45*	−1.18*	−1.52*	−1.20*
	(−5.13)	(−4.29)	(−7.05)	(−4.32)
MICRO	0.20	0.55	−0.39	0.56
	(0.48)	(1.36)	(−1.22)	(1.37)
n	240	229	240	115
LL	−84.2	−213.6	−122.7	−54.8

Notes: Numbers in parentheses are *t*-ratios. *(‡)[†] denote the 0.01(0.05)[0.10] levels of significance, respectively.

One clear limitation in Smyth and Kroncke (2005), and in the empirical approach taken above, is in the experimental design. Future research should control for the treatment by administering the algebra quiz to students in another course (e.g., principles of marketing) so that any improvements in math skills exhibited by the students in the economics course can be attributed to economic education, and not to some other factor. By controlling for a wide range of covariates, as is done in this study and in Smyth and Kroncke (2005), a positive and significant coefficient estimate attached to a binary variable that is equal to 1 for students enrolled in the economics course, and 0 otherwise, would provide more compelling evidence that taking an economics course can actually improve math skills.

Despite the aforementioned limitation, our work argues in support of the efforts articulated by Ganter and Harver (2011) for adding more practical context to the instruction in mathematics and agree with Wible (2009) when highlighting the role of mathematics as one, among several, of the reasoning tools serving the broad analytical work of economics. Even within a traditional curriculum organized around discrete courses, a closer partnering between disciplines can yield positive synergies: mathematics would benefit from using economics as a means of instruction and economics would benefit from making explicit use of mathematics as a mean to conduct formal analysis, being careful not to present it as a sublimation of economics itself.

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Appendix: The Math Skills Quiz

***Traditional* Algebra Questions**

Question 2. Suppose that $z = x/y$, and you know that $z = 5$ and $x = 15$. Find the value of y.

a. 15 b. 75 c. 3 d. 1/3

Question 3. Perform the following division: (1/6) ÷ (2/3)

a. 9 b. ¼ c. 1/9 d. 4

Question 6. Suppose the value of X is 6. Find the absolute value of X.

a. −6 b. 6 c. 1/6 d. not enough information given to answer

Question 9. Suppose the value of Y is −8. Find the absolute value of Y.

a. 1/8 b. −8 c. 8 d. not enough information given to answer

Question 10. Solve the following equation for Q: Q = 48 + (1/4)Q

a. 64 b. 36 c. 16 d. 12

***Word-problem* Algebra Questions**

Question 1. Suppose you see a shirt in a store that costs $50. You return the next week and find that the shirt has been marked down on sale to $30. What is the percentage saved on the price of the shirt (i.e., what percentage is the sale)?

a. 60% b. 66.67% c. 33.33% d. 40%

Question 4. Consider two variables, X and Y. If you observe that X decreases from a value of 13 to 9, and in response, Y increases from a value of 2 to 4, then we say that X and Y have:

a. a negative relationship
b. a positive relationship
c. a neutral relationship
d. there is not enough information given to determine their relationship

Question 7. Suppose you want to carpet a room that is 12 feet long and 8 feet wide. The carpet you've chosen costs $2 per square foot. How much will it cost to purchase enough carpet?

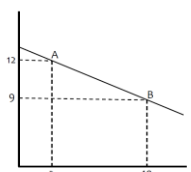
a. $96 b. $192 c. $40 d. $48

Question 11. Suppose a bakery receives an order for three containers of their very special cookies. Each container will have 20 cookies in it, and the total cost of producing enough cookies to fill a single container is $40. What is the bakery's average production cost per cookie?

a. $0.50 b. $6 c. $0.67 d. $2

***Graph-based* Algebra Questions**

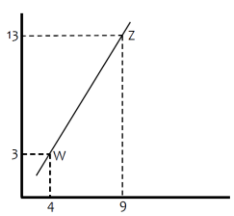
Question 5. Consider the graph below.



The coordinates to point A are (2,12) and the coordinates of point B are (10,9). Find the slope of the line.

a. 3/8 b. −3/8 c. 8/3 d. −8/3

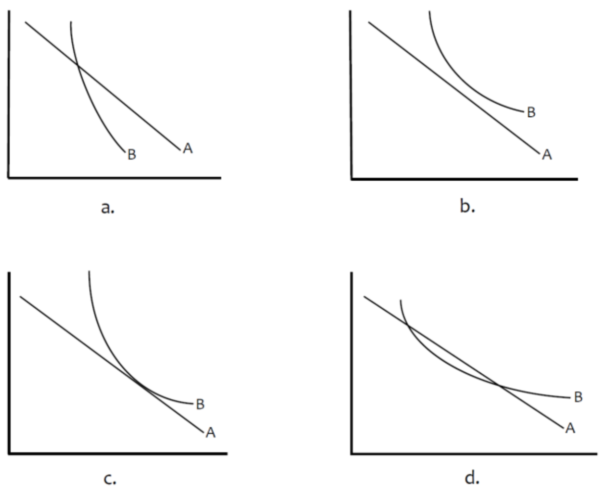
Question 8. Consider the graph below.



The coordinates of point *W* are (4,3) and the coordinates of point *Z* are (9,13). Find the slope of the line.

a. ½ b. −2 c. 10 d. 2

Question 12. Which of the graphs on the following page shows a point of tangency between line A and curve B?



Source: Smyth and Kroncke (2005).